Versuchsvorbereitung -

Ausführungen zum Punkt 3

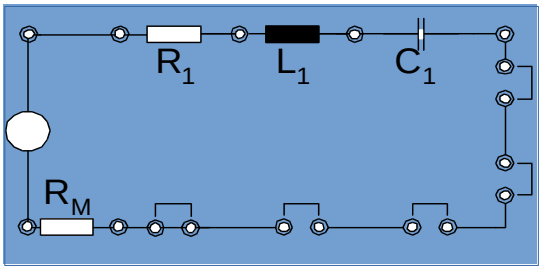
Grundlagen und Ermittlung der wichtigsten Kennwerte von Resonanzkreisen und Passschaltungen

3.1.1 C_1=**50µF**

f/Hz	Z/Ω	I (f)/mA	I (f)/I (f_0)	U_L/V	U_L/U_g	U_C/V	U_C/U_g
f0							

f0, f±45, 3f0, f0/3, 6f0, f0/6, 10f0, f0**/10**

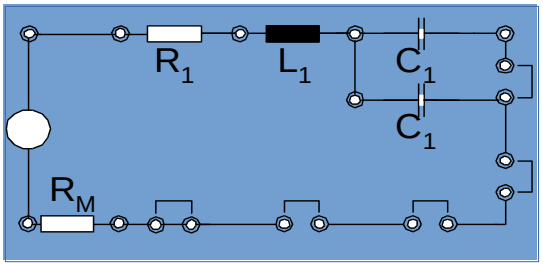
$$Z(f) = R + j\left(2\pi f L_1 - \frac{1}{2\pi f C_1}\right)$$



3.1.2 C_1=**100µF**

f/Hz	Z/Ω	I (f)/mA	I (f)/I (f_0)	U_L/V	U_L/U_g	U_C/V	U_C/U_g
f0							

f0, f±45, 3f0, f0/3, 6f0, f0/6, 10f0, f0/10

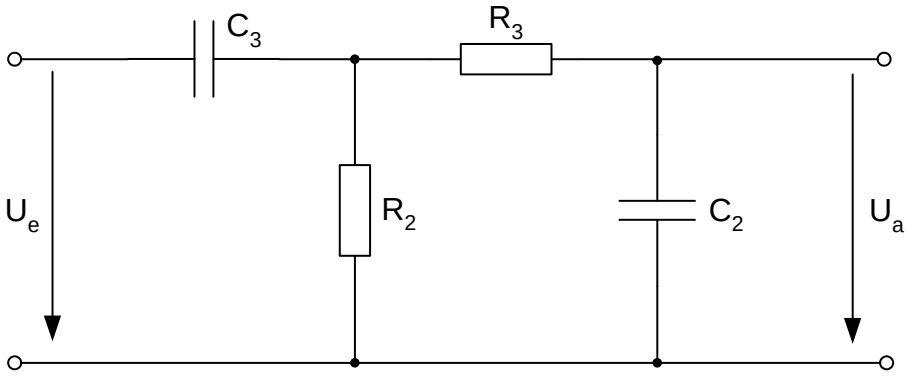


3.1.3 C_1=**50µF** **Bandbreite von 100 Hz**

$$b_f = f_{+45°} - f_{-45°} = \left(\frac{R}{2L} + \frac{R}{2L}\right)\frac{1}{2\pi} = \frac{R}{L}\frac{1}{2\pi} = 100\,Hz$$

f/Hz	Z/Ω	I (f)/mA	I (f)/I (f_0)	U_L/V	U_L/U_g	U_C/V	U_C/U_g
f-45							
f0							
f+45							

3.2 Bandpass (10 Hz < f_g < 50 kHz)

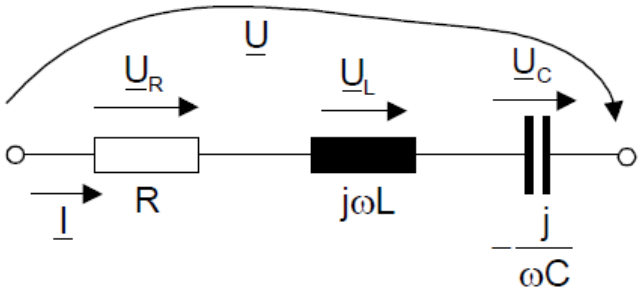


Bauelemente: R_2 = 100 Ω, R_3 = 100 Ω, C_2 = 1 µF, C_3 = 10 µF;
Berechnen Sie die Übertragungsfunktion $\underline{U_a}/\underline{U_e}$.

	f/Hz	U_e/V	U_a/V $_{Mess}$	U_a/V $_{Oszi}$
3.2.2				
	10	1		
	50	1		
	100	1		
	150	1		
	250	1		
	350	1		
	500	1		
	750	1		
	1000	1		
	1500	1		
	5000	1		
	10000	1		
	20000	1		
	30000	1		
	50000	1		

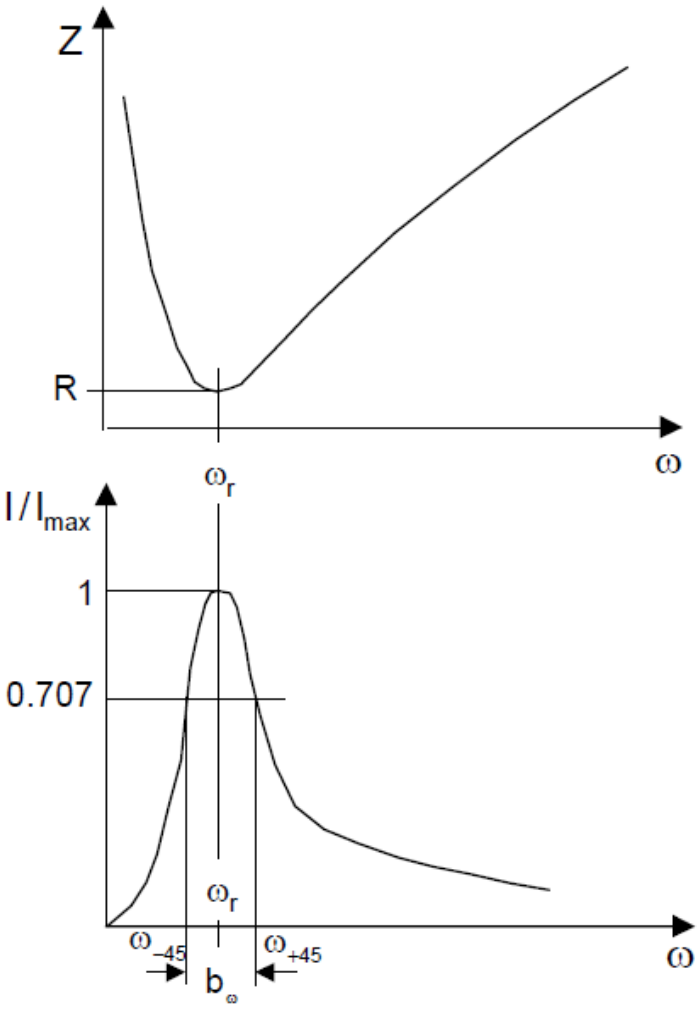
Oszillografieren Sie die Eingangsspannung am Kanal CH_1 und die Ausgangsspannung am Kanal CH_2 bei f_0 !

Übersicht 1

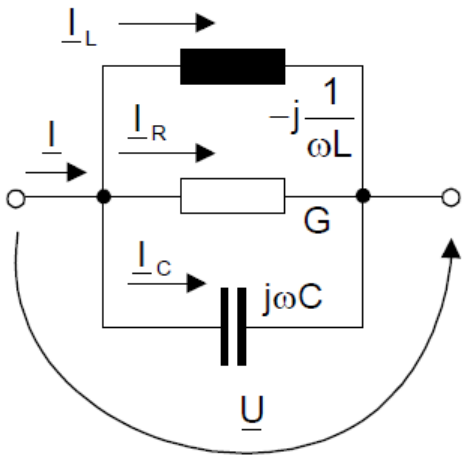


$$\underline{Z}(\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z(\omega) = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

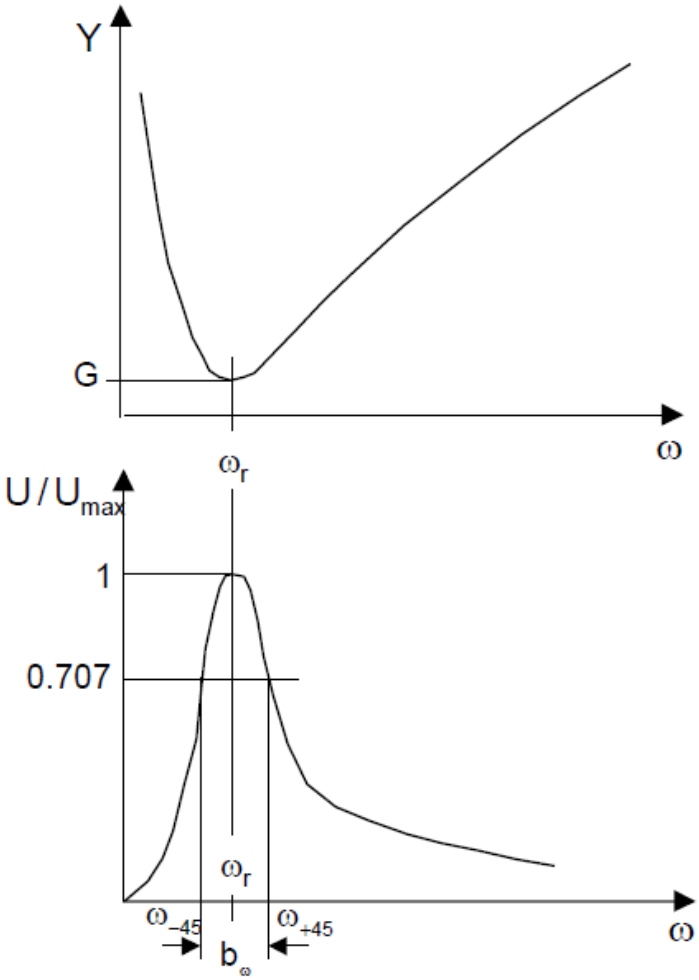


Übersicht 2

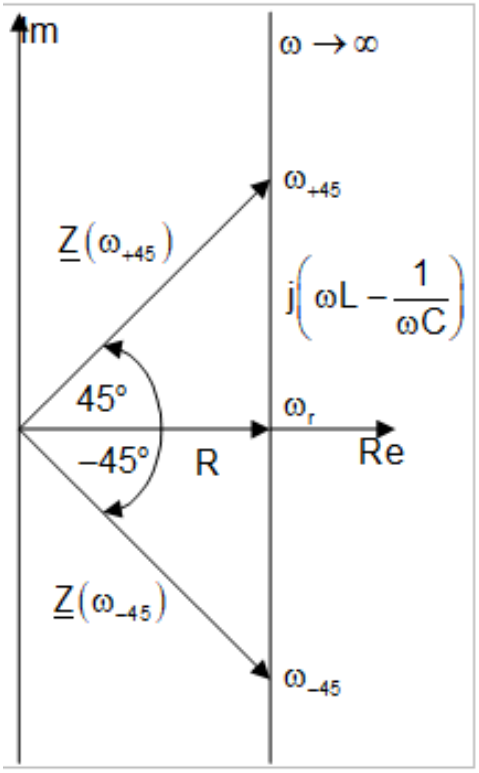


$$\underline{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$Y(\omega) = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$



Übersicht 3



$$\underline{Z}(\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Definition der Resonanzfrequenz ω_r:

$\mathrm{Im}\{\underline{Z}(\omega_r)\} = 0$

$$\omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$ Thomsonsche Formel

Definition der 45°-Frequenzen

$Z(\omega_{-45}) = Z(\omega_{+45}) = \sqrt{2}R$

$I(\omega_{-45}) = I(\omega_{+45}) = \frac{1}{\sqrt{2}} I_{max} = 0.707 \cdot I_{max}$

$$R = \left(\omega_{+45} L - \frac{1}{\omega_{+45} C}\right) \qquad \omega_{+45} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$R = \left(\frac{1}{\omega_{-45} C} - \omega_{-45} L\right) \qquad \omega_{-45} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Übersicht 4

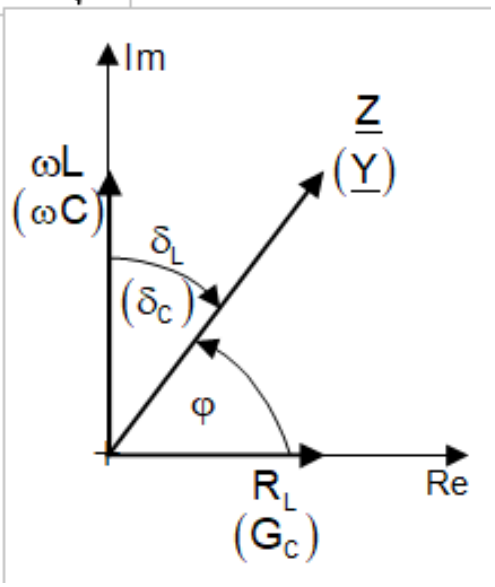
Bandbreite: $b_\omega = \omega_{+45} - \omega_{-45} = \frac{R}{L}$

$b_f = f_{+45} - f_{-45} = b_\omega / 2\pi$

Resonanzüberhöhung: $\rho = \frac{U_L(\omega_r)}{U} = \frac{U_C(\omega_r)}{U}$

$\rho = \frac{\omega_r L}{R} = \frac{1}{R \cdot \omega_r C} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$

Verlustwinkel: $\tan\delta = \frac{R}{\omega_r L} = R \cdot \omega_r C = \frac{1}{\rho}$



Bandbreite: $b_\omega = \omega_{+45} - \omega_{-45} = \frac{1}{RC}$

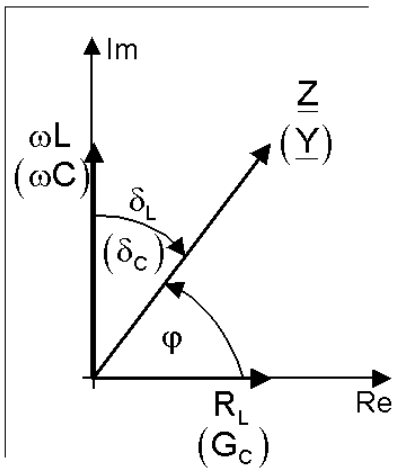
$b_f = f_{+45} - f_{-45} = b_\omega / 2\pi$

Resonanzüberhöhung: $\rho = \frac{I_L(\omega_r)}{I} = \frac{I_C(\omega_r)}{I} = g$

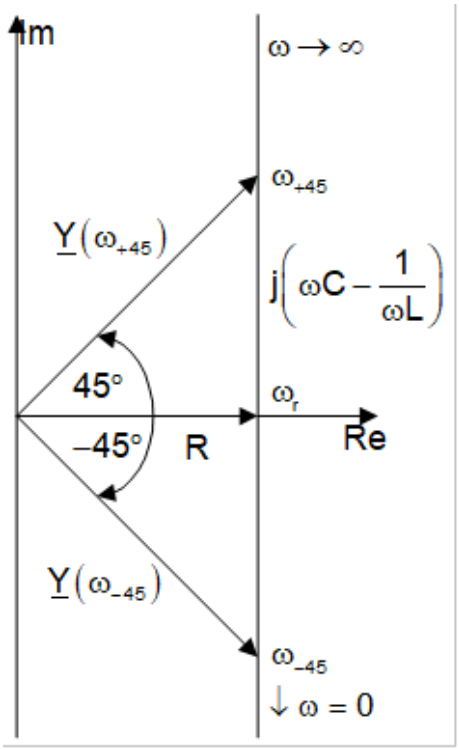
$$\rho = \frac{R}{\omega_r L} = R \cdot \omega_r C = \frac{R}{L} \cdot \frac{\sqrt{LC}}{1} = R \cdot \sqrt{\frac{C}{L}}$$

Verlustwinkel: $\tan\delta = \frac{\omega_r L}{R} = \frac{1}{R \cdot \omega_r C} = \frac{1}{\rho} = d$

(Dämpfungsgrad)



Übersicht 6



$$\underline{Y}(\omega) = \underline{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

Definition der Resonanzfrequenz ω_r:

$\mathrm{Im}\{\underline{Y}(\omega_r)\} = 0$

$$\omega_r C - \frac{1}{\omega_r L} = 0$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$ Thomsonsche Formel

Definition der 45°-Frequenzen:

$$Y(\omega_{-45}) = Y(\omega_{+45}) = \frac{\sqrt{2}}{R}$$

$$U(\omega_{-45}) = U(\omega_{+45}) = \frac{U_{max}}{\sqrt{2}} = 0.707\, U_{max}$$

bei den 45°-Frequenzen sind Real und Imaginärteil des Leitwertoperators gleich.

$$\frac{1}{R} = \omega_{+45} C - \frac{1}{\omega_{+45} L}$$

$$\omega_{+45} = \frac{1}{2 \cdot RC} + \sqrt{\left(\frac{1}{2 \cdot RC}\right)^2 + \frac{1}{LC}}$$

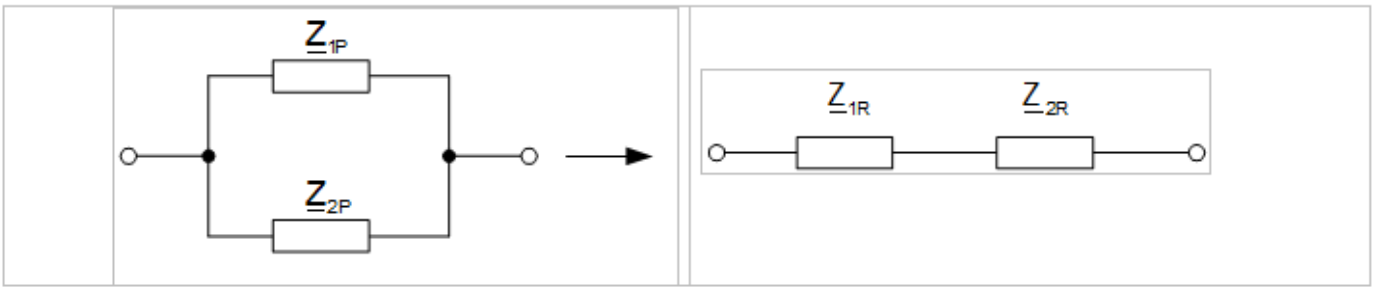
$$\frac{1}{R} = \frac{1}{\omega_{-45} L} - \omega_{-45} L$$

$$\omega_{-45} = -\frac{1}{2 \cdot RC} + \sqrt{\left(\frac{1}{2 \cdot RC}\right)^2 + \frac{1}{LC}}$$

Übersicht 7

Umwandlung Reihen- und Parallelschaltungen

Beide Schaltungen liefern den gleichen Gesamtwiderstandsoperator



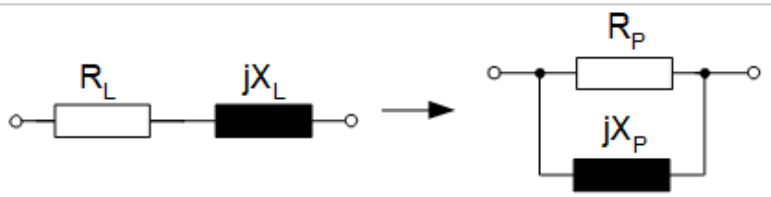
$\underline{Z}_{1P} = R_P = 100\Omega$ $\qquad$ $\underline{Z}_{2P} = -jX_C = -j150\Omega$

$$\underline{Z}_g = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{R \cdot (-jX_C)}{R - jX_C} = \frac{-j100\Omega \cdot 150\Omega}{100\Omega - j150\Omega} = \frac{15000\Omega^2 \underline{|-90^\circ}}{180.3\Omega \underline{|-56.3^\circ}}$$

$$\underline{Z}_g = 83.2\Omega \underline{|-33.7^\circ} = \underline{69.2\Omega} - \underline{\underline{j46.2\Omega}}$$

$\underline{Z}_{1R} = R_R = \underline{69.2\Omega}$ $\qquad$ $\underline{Z}_{2R} = -jX_{CR} = \underline{\underline{-j46.2\Omega}}$

Liegt eine Reihenschaltung vor, die in eine äquivalente Parallelschaltung umgewandelt werden soll, dann wird aus $\underline{Z}$ der Leitwertoperator $\underline{Y}$ gebildet



$$\underline{Z} = R_L + jX_L = 11\Omega + j5.86\Omega$$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R_L + jX_L} = \frac{1}{11\Omega + j5.86\Omega} \qquad \underline{Y} = G_P - jB_P = \underline{70.8mS} - \underline{\underline{j37.7mS}}$$

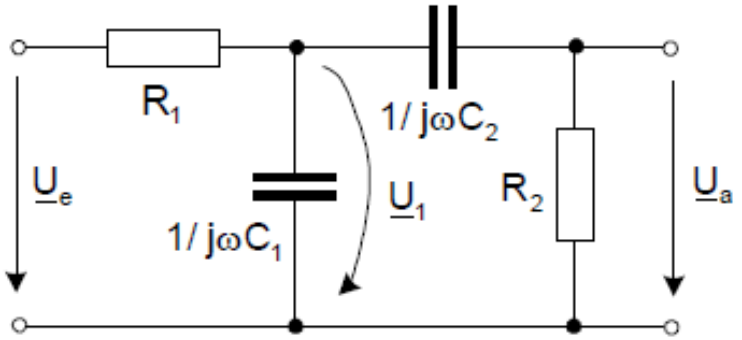
$$R_P = \frac{1}{G_P} = \frac{1}{\underline{70.8mS}} = 14.1\Omega \qquad X_P = \frac{1}{B_P} = \frac{1}{\underline{\underline{37.7mS}}} = 26.5\Omega$$

$\underline{Z}_g$ = $\underline{69.2\Omega}$ - $\underline{\underline{j\,46.2\Omega}}$= $83.2\Omega\angle{-33.7^\circ}$ $\underline{Y}_g$ = 12.0 mS$\angle 33.7^\circ$ = 10 mS + j 6.67 mS

$$\underline{Z}_{1P} = \frac{1}{\underline{Y}_{1P}} = \frac{1}{10mS} = 100\Omega \qquad \underline{Z}_{2P} = \frac{1}{\underline{Y}_{2P}} = \frac{1}{j6.67mS} = -j150\Omega$$

Übersicht 9

Beispielaufgabe zur Berechnung eines Bandpasses:



$R_1 = R_2 = R = 1k\Omega;$
$C_1 = C_2 = C = 3.185\mu F$

$$X_{C1} = X_{C2} = X_C = \frac{1}{\omega C}$$

a)

$$\frac{\underline{U}_a}{\underline{U}_1} = \frac{R_2}{R_2 - jX_{C2}} = \frac{R}{R - jX_C}$$

$$\underline{Z} = (R_2 - jX_{C2}) \| (-jX_{C1}) = \frac{(R - jX_C)\cdot(-jX_C)}{R - j2X_C}$$

$$\frac{\underline{U}_1}{\underline{U}_e} = \frac{\underline{Z}}{R_1 + \underline{Z}} = \frac{\dfrac{(R - jX_C)\cdot(-jX_C)}{R - j2X_C}}{R + \dfrac{(R - jX_C)\cdot(-jX_C)}{R - j2X_C}} = \frac{(R - jX_C)(-jX_C)}{R(R - j2X_C) + (R - jX_C)(-jX_C)}$$

$$\frac{\underline{U}_a}{\underline{U}_e} = \frac{\underline{U}_a}{\underline{U}_1}\cdot\frac{\underline{U}_1}{\underline{U}_e} = \frac{R}{(R - jX_C)}\cdot\frac{(R - jX_C)(-jX_C)}{R(R - j2X_C) + (R - jX_C)(-jX_C)}$$

$$\frac{\underline{U}_a}{\underline{U}_e} = \frac{(-jRX_C)}{R(R - j2X_C) + (R - jX_C)(-jX_C)} = \frac{-jRX_C}{R^2 - j3RX_C - X_C^2}$$

$$\frac{\underline{U}_a}{\underline{U}_e} = \frac{1}{\dfrac{R^2}{-jRX_C} - \dfrac{j3RX_C}{-jRX_C} - \dfrac{X_C^2}{-jRX_C}} = \frac{1}{\dfrac{jR}{X_C} + 3 - j\dfrac{X_C}{R}}$$

$$\boxed{\underline{H} = \frac{\underline{U}_a}{\underline{U}_e} = \frac{1}{3 + j\left(R\omega C - \dfrac{1}{R\omega C}\right)}}$$

Übersicht 10

Konkrete Vorgabewerte: siehe Versuchsanleitung!