



Heat Exchanger

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1 INTRODUCTION

Technical processes are widely used in the industry. Due to the characteristic features, the mathematical description of the static and dynamic behavior and the technical realization of it form a certain class of technological procedures. The simulation and the design of technical systems will be discussed on the example of technical heating process problems.

The aims of this experiment are:

1. To develop a mathematical model with immeasurable disturbances on the basis of a theoretical process analysis.
2. To transform the model into the simulation program Matlab and make experiments which determine the parameters of the real process.
3. To investigate the properties of the model through several simulation experiments. The dynamic behavior of the process is available and measured for the verification of the real process.
4. The focus of the programming work is to the development of Simulink programs and to using and comparing of the transfer functions and linear state equations. The dynamic system is described by:
 - a. two transfer functions,
 - b. the system of state equations.

2 DESCRIPTION OF THE EXPERIMENTAL EQUIPMENT

A functional diagram of the laboratory test rig with a signal representation is shown in Fig. 1. The process consists of the boiler (the water tank), the electrical heating unit, the power control unit, couple of thermometers, some converters, the cooling system, two auxiliary water tanks, two pumps and a PC with a measurement I/O-card (see Tab. 1).

1. The main part is the boiler. Water flows through the open boiler (PO) with the mass flow $\dot{m} = M$. The mass flow is controlled by the water pumps (C1, C2). The temperature of water at the inlet of the boiler is $T_E(t)$, the temperature at the outlet is $T(t)$.
2. The electrical heating unit (TS) is used to heat water inside the boiler. The electrical output of the heating unit is controlled by the electrical power control unit (PWM). This device is driven by an external voltage u_0 in the range of 0 to 10 volts. The electrical power at the output of the device is changed in the range of 0 to 760 W. The water temperature inside the boiler is measured by the Pt100 type resistance thermometer (CT) and converted with an R/U converter into a voltage u_T .
3. After heating, water is cooled down in a cooling system and optionally flows into the main water tank (HB1) or into the auxiliary water tank (HB2). There are water pumps inside the tanks. The water pumps are independently controlled by the electric power source (ZD1) and together provide the mass flow \dot{m} .
4. All signals are connected to the measurement I/O-card (MK). The data acquisition software is installed on the PC.

Tab. 1: Description of components

PO	Boiler	HB1, HB2	Water tank
TS	Electrical heating unit	C1	Water pump
PWM	Power control unit	C2	Water pump (disturbance)
R/U	R/U converter	MK	Measurement I/O-card
CT	Temperature sensor	AO0	Analog output
ChS	Cooling system	AI0	Analog input

Tab. 2: Signal description

Signal	Description	Range
u	Matlab output	-1 .. 1
u_0	Measurement I/O-card output	0 .. 10 V
u_p	Heater power voltage	0 .. 230 V
u_T	Measurement I/O-card input	0 .. 10 V
y	Matlab input	0 .. 1

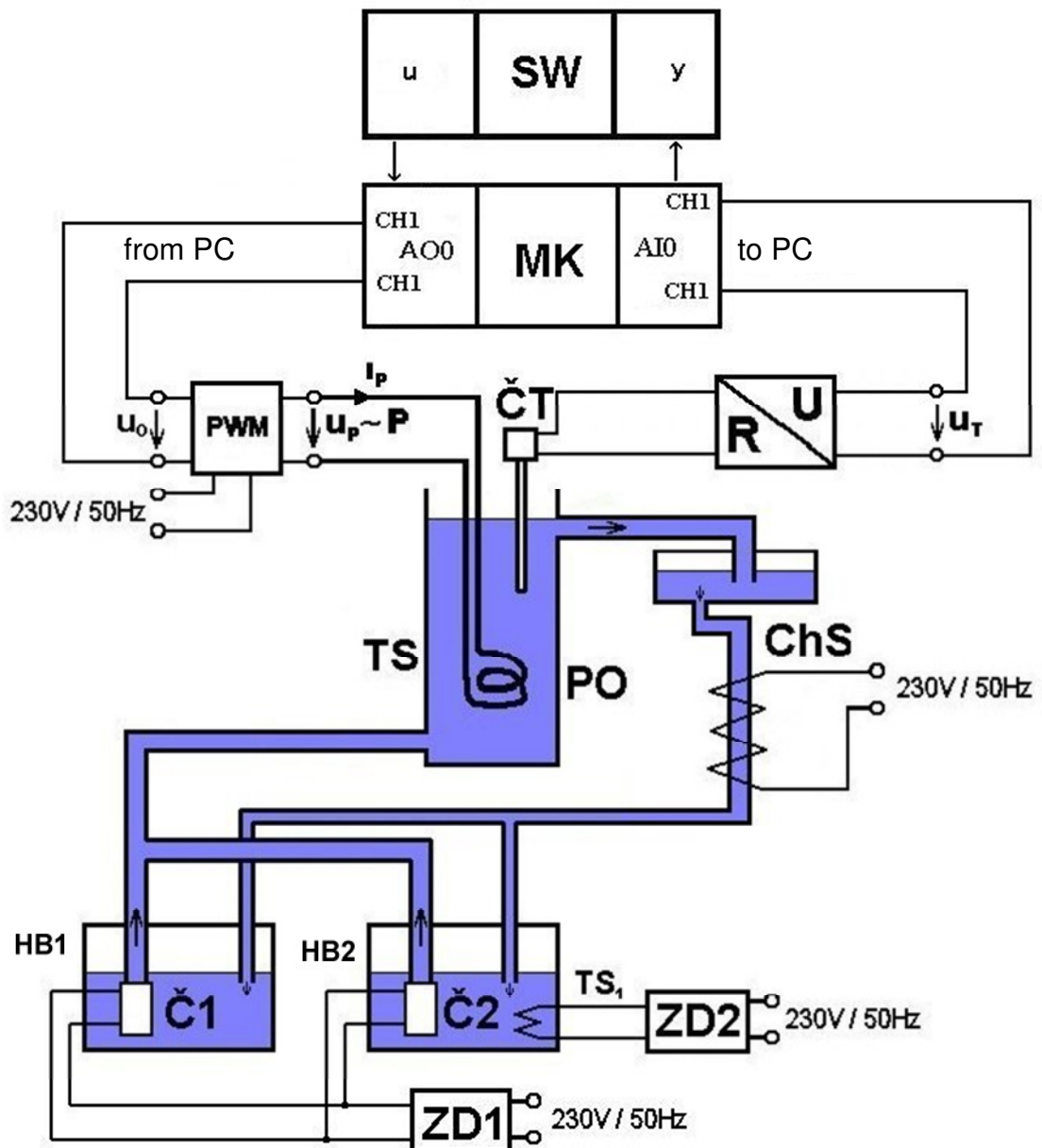


Fig. 1: Block diagram of the process

3 THEORETICAL PROCESS ANALYSIS

The 1st order differential equation system will be created from the theoretical process analysis. The analysis is based on the mass and energy balance. These equations directly built the state description in the time domain.

3.1 SYSTEM SIMPLIFICATIONS

The mathematical model will be performed for the part of the plant, which is shown in Fig. 2. Input variable is the power of the heating unit (TS) P in W. Water flows through the plant with the mass flow M in kg/s and with the inlet temperature T_{IN} in °C. The outlet water temperature T_{OUT} is measured with the temperature sensor (CT) in °C.

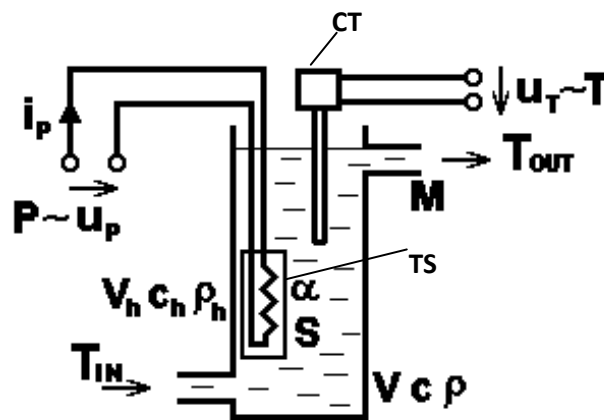


Fig. 2: Boiler subsystem

Tab. 3: Boiler subsystem physical values

Symbol	Description	Unit	Symbol	Description	Unit
T_{IN}	Inlet temperature	°C	T_h	Heater unit temperature	°C
T_{OUT}	Outlet temperature	°C	V_h	Heater unit volume	m ³
M	Water mass flow	kg·s ⁻¹	c_h	Heater heat capacity	J·kg ⁻¹ ·K ⁻¹
V	Boiler water volume	m ³	ρ_h	Heater density	kg·m ⁻³
c	Water heat capacity	J·kg ⁻¹ ·K ⁻¹	α	Heat transfer coefficient	W·m ⁻² ·K ⁻¹
ρ	Water density	kg·m ⁻³	S	Heater unit area	m ²

Assumptions for the mathematical and physical modeling are in this case:

1. The system is considered as a system with concentrated parameters.
2. Ideal heat insulation of the container.
3. Constant mass flow (water flow).
4. Ideal mixture of the water inside the boiler.
5. The electrical performance of the radiator is converted into heat energy without any losses.

These assumption leads to the idealized system, which is shown in Fig. 3.

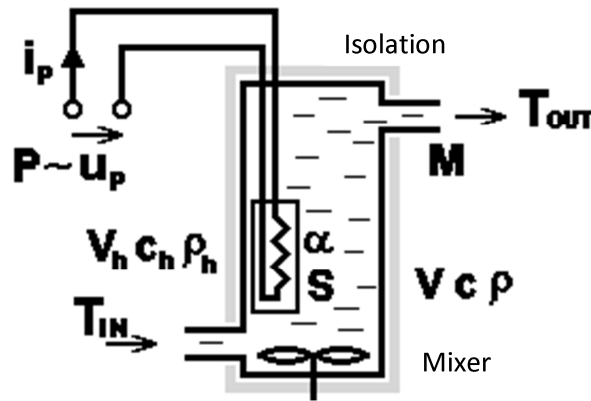


Fig. 3: Idealized boiler subsystem

3.2 HEAT BALANCE

Under the above conditions one can write the heat balance in the following form

$$\left[\begin{array}{c} \text{Time changes} \\ \text{of stored heating energy} \end{array} \right] = \left[\begin{array}{c} \text{input} \\ \text{heating energy} \end{array} \right] - \left[\begin{array}{c} \text{output} \\ \text{heating energy} \end{array} \right]. \quad (1)$$

This corresponds to the equation

$$\frac{dW(t)}{dt} = P_{IN} - P_{OUT}, \quad (2)$$

where $W(t)$ is the stored heat energy (in J), P_{IN} is the input heating power (in W) and P_{OUT} is the overall output heating power (in W).

The stored heat energy is calculated for the heater body and for the water inside the boiler.

The energy in the water can be formulated as

$$W_w = W_w(t) = c \cdot \rho \cdot V \cdot T_{OUT}(t) \quad (3)$$

and the energy stored in the heater results in

$$W_h = W_h(t) = c_h \cdot \rho_h \cdot V_h \cdot T_h(t). \quad (4)$$

The input heat power for the heater can be computed as a heat transferred inside by the boiler by the inflow water plus the electric power of the heater $P(t)$ in W. The output heat power is the heat transferred to the outflowing water.

$$\begin{aligned} P_{IN,h}(t) &= \alpha \cdot S \cdot T_{OUT}(t) + P(t) \\ P_{OUT,h}(t) &= \alpha \cdot S \cdot T_h(t) \end{aligned} \quad (5)$$

Now the heat balance for the heater can be rewritten by substituting equations (3), (4) and (5) to equation (2) into the form

$$\frac{d}{dt} [c_h \cdot \rho_h \cdot V_h \cdot T_h(t)] = \alpha \cdot S \cdot [T_{OUT}(t) - T_h(t)] + P(t). \quad (6)$$

Similarly, the flowing water goes inside the boiler with a heat (depending on the actual temperature and flow rate) and moreover the heat from the heater must be added, too. On the output side, the heat is going out with the water and some part of heat is transferred to the output water by conduction.

$$P_{IN,w}(t) = \alpha \cdot S \cdot T_h(t) + M \cdot c \cdot T_{IN}(t) \quad (7)$$

$$P_{OUT,w}(t) = \alpha \cdot S \cdot T_{OUT}(t) + M \cdot c \cdot T_{OUT}(t)$$

Next, the heat balance for the water volume is given by substituting equations (3) and (7) to equation (2) in the following form

$$\frac{d}{dt}[c \cdot \rho \cdot V \cdot T_{OUT}(t)] = M \cdot c \cdot [T_{IN}(t) - T_{OUT}(t)] + \alpha \cdot S \cdot [T_h(t) - T_{OUT}(t)]. \quad (8)$$

Finally, two differential equation sets for the temperatures of the output water and the heater body can be formulated from equations (6) and (8).

$$\frac{d}{dt}T_h(t) = -\frac{\alpha S}{c_h \rho_h V_h}T_h(t) + \frac{\alpha S}{c_h \rho_h V_h}T_{OUT}(t) + \frac{P(t)}{c_h \rho_h V_h} \quad (9)$$

$$\frac{d}{dt}T_{OUT}(t) = \frac{\alpha S}{c \rho V}T_h(t) - \frac{\alpha S + Mc}{c \rho V}T_{OUT}(t) + \frac{M}{\rho V}T_{IN}(t) \quad (10)$$

The differential equation system contains two input functions $P(t)$ and $T_{IN}(t)$. The manipulated (control) variable is the electrical power $u(t) = P(t)$, which can be changed by the actuator. The second input function is the inlet temperature of the water $T_{IN}(t)$, which cannot be influenced in this case and is not measured. Therefore, this input is considered as an immeasurable disturbance $d(t) = T_{IN}(t)$.

3.3 STEADY-STATES

For the steady state it is required that all derivatives with respect to the time are equal to zero. The following condition must be met

$$\lim_{t \rightarrow \infty} \frac{d}{dt}T_h(t) = \lim_{t \rightarrow \infty} \frac{d}{dt}T_{OUT}(t) = 0. \quad (11)$$

Using this formula to equations (9) and (10) it is possible to get

$$T_h(\infty) = T_{OUT}(\infty) + \frac{1}{\alpha S}P(\infty) \quad (12)$$

$$T_{OUT}(\infty) = \frac{\alpha S}{\alpha S + Mc}T_h(\infty) + \frac{Mc}{\alpha S + Mc}T_{IN}(\infty) \quad (13)$$

The final solution of the steady state problem is covered in equations (12) and (13). It is possible to obtain the following steady state values for both temperatures after solving of these equations set with a respect to the input variables $P(t)$ and $T_{IN}(t)$

$$T_h(\infty) = T_{IN}(\infty) + \frac{\alpha S + Mc}{\alpha S \cdot Mc}P(\infty) \quad (14)$$

$$T_{OUT}(\infty) = T_{IN}(\infty) + \frac{1}{Mc}P(\infty) \quad (15)$$

3.4 DERIVATION OF THE DIFFERENTIAL EQUATION

It is assumed that the process is described by the state equations (9) and (10). For simplicity, we introduce the following designation

$$y = T_{OUT}, \quad x = T_h, \quad u = P, \quad d = T_{IN},$$

$$a_{11} = \frac{\alpha S}{c_h \rho_h V_h}, \quad a_{21} = \frac{\alpha S}{c \rho V}, \quad a_{22} = \frac{\alpha S + Mc}{c \rho V}, \quad b_{11} = \frac{1}{c_h \rho_h V_h}, \quad b_{22} = \frac{M}{\rho V}.$$

Since the electrical power $P(t)$ can be selectively changed by the power control unit, this represents a (manipulated) control value $u(t)$. The water temperature $T_{IN}(t)$ cannot be precisely adjusted, but it affects the output value $y(t)$, the temperature $T_{OUT}(t)$. Thus $T_{IN}(t)$ represents a (immeasurable) disturbance $d(t)$. The heater temperature represents an (immeasurable) inner state of the process $x(t)$.

Therefore the differential equations (6) and (8) can be rewritten in the following form

$$\dot{x}(t) = -a_{11}x(t) + a_{11}y(t) + b_{11}u(t), \quad (16)$$

$$\dot{y}(t) = a_{21}x(t) - a_{22}y(t) + b_{22}d(t). \quad (17)$$

The equations (17) can be written with the state variable on the one side in the form

$$x(t) = \frac{1}{a_{21}} [\dot{y}(t) + a_{22}y(t) - b_{22}d(t)] \quad (18)$$

The task is to find the mathematical model for the calculation of the outlet water temperature from the boiler. For this it is first necessary to eliminate the temperature of the heater from the equations (16) and (17). The elimination method is used. Firstly, the equation (17) is derived.

$$\ddot{y}(t) = a_{21}\dot{x}(t) - a_{22}\dot{y}(t) + b_{22}\dot{d}(t) \quad (19)$$

Next, in the equation (19) the equation (16) can be used for substituting the $\dot{x}(t)$. The variable $x(t)$ is calculated from the equation. The following process leads to the final 2nd order differential equation (20).

$$\ddot{y}(t) = a_{21} \cdot [-a_{11}x(t) + a_{11}y(t) + b_{11}u(t)] - a_{22}\dot{y}(t) + b_{22}\dot{d}(t)$$

$$\ddot{y}(t) = a_{21} \cdot \left[-a_{11} \frac{1}{a_{21}} [\dot{y}(t) + a_{22}y(t) - b_{22}d(t)] + a_{11}y(t) + b_{11}u(t) \right] - a_{22}\dot{y}(t) + b_{22}\dot{d}(t)$$

$$\ddot{y}(t) = -(a_{11} + a_{22})\dot{y}(t) - (a_{11}a_{22} - a_{21}a_{11})y(t) + a_{21}b_{11}u(t) + a_{11}b_{22}d(t) + b_{22}\dot{d}(t)$$

$$\ddot{y}(t) + a_1\dot{y}(t) + a_0y(t) = b_0u(t) + c_0d(t) + c_1\dot{d}(t) \quad (20)$$

where $a_1 = (a_{11} + a_{22})$, $a_0 = (a_{11}a_{22} - a_{21}a_{11})$, $b_0 = a_{21}b_{11}$, $c_0 = a_{11}b_{22}$, $c_1 = b_{22}$.

3.5 TRANSFER FUNCTION

The application of the \mathcal{L} -transformation to the differential equation (20) with zero initial conditions results in the following \mathcal{L} -domain equation

$$Y(s)(s^2 + a_1s + a_0) = b_0U(s) + (c_0 + c_1s)D(s) \quad (21)$$

where $Y(s) = \mathcal{L}\{y(t)\}$, $U(s) = \mathcal{L}\{u(t)\}$, $D(s) = \mathcal{L}\{d(t)\}$.

The \mathcal{L} -image of the output signal is equal to

$$Y(s) = \frac{b_0}{s^2 + a_1s + a_0} U(s) + \frac{c_1s + c_0}{s^2 + a_1s + a_0} D(s) = F_u(s) \cdot U(s) + F_d(s) \cdot D(s) \quad (22)$$

where

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$F_u(s) = \frac{b_0}{s^2+a_1s+a_0}$ is the transfer function describing the dynamic effect of the manipulated (control) variable $u(t)$ with respect to output $y(t)$ and

$F_d(s) = \frac{c_1s+c_0}{s^2+a_1s+a_0}$ is the transfer function describing the dynamic effect of the disturbance $d(t)$ with respect to output $y(t)$.

The model structure of the dynamical system with the control and disturbance inputs is shown in Fig. 4.

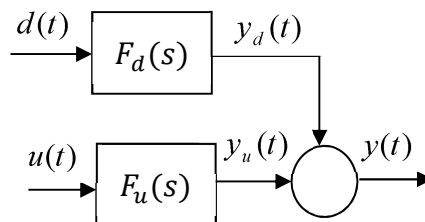


Fig. 4: The dynamic process model

4 SIMULATION

4.1 INPUT AND OUTPUT SIGNAL CONVERSION

The mathematical model in Simulink is excited by the electrical heating power $P(t)$. If the measured input into the model is realized via the block 'From Workspace', the signal $u(t)$ must be converted into the physical variable $P(t)$. The conversion formula is

$$P(t) = 380 \cdot u(t) + 380 \quad (23)$$

The output signal from the model is equal to the water temperature $T_{OUT}(t)$. It is compared with the measured signal $y(t)$, which is used with the help of the block 'From Workspace' as a data source. The following conversion has to be applied.

$$y(t) = \frac{1}{100} T_{OUT}(t) \quad (24)$$

The verification is done subjectively by comparing the real output of the process and the outputs from the model in one graph.

4.2 SIMULATION PARAMETERS

The simulation should be performed with the parameters that are given in Tab. 4 and Tab. 5.

Tab. 4: Parameters of the heat exchanger

Parameter	ρ	c	V	M	T_{IN}
Unit	$\text{kg}\cdot\text{m}^3$	$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$	m^3	$\text{kg}\cdot\text{s}^{-1}$	$^{\circ}\text{C}$
Value	1000	4186	$149.1\cdot 10^{-6}$	$5.17\cdot 10^{-3}$	24

Tab. 5: Parameters of the heater

Parameter	ρ_h	c_h	V_h	S	α
Unit	$\text{kg}\cdot\text{m}^3$	$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$	m^3	m^2	$\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$
Value	7800	452	$1.965\cdot 10^{-6}$	$17\cdot 10^{-3}$	800

5 TASKS

5.1 THEORETICAL PROCESS ANALYSIS

Perform a theoretical process analysis and explain the differential equations that are given in this manual. Derive the transfer function

1. $F_u(s) = \frac{Y(s)}{U(s)} = \frac{B(s)}{A(s)}$ and
2. $F_d(s) = \frac{Y(s)}{D(s)} = \frac{C(s)}{A(s)}$.

5.2 CREATING OF THE MODEL USING MATLAB / SIMULINK

The goal is to create a model in the Matlab environment. In principle, a simulation program is written in the Simulink (mdl-file), but is controlled by a Matlab program (m-file). This means that all the necessary parameters calculations and program instructions are programmed in the Matlab.

- For the model verification there is the file 'mer02.mat' for the flow rate 310 ml/min prepared. This file does not contain any control in the closed loop.
- The parameters for the simulations are shown in Tab. 4 and Tab. 5. For parameters which are not present or cannot be measured choose reasonable defaults and vary them to reach the measured outputs. **Optimize them!**

The aim is to minimize the deviations between model and real system!

- Follow next steps to construct the simulation model of the heater in several forms and verify the dynamic behavior and steady states of all models.

5.2.1 Task 1 – data preprocessing

Write a Matlab programs (m-file) which contains all parameter calculations and which gives the graphical output to the graph.

- Load the file 'mer02.mat'. This file contains a variable named 'ans'. The first line in this variable contains the time t , the second is the controlled variable y , the third is the manipulated variable u . All variables are in 'Matlab' ranges.
- Use the function 'plot' to show the course of variables t , u , y in 'Matlab' ranges.
- Construct the second plot with all variables in their physical units! Use Tab. 2 and equations (23) and (24).
- Explain the measurement.

5.2.2 Task 2 – steady state and first verification

Calculate the steady state temperatures for the flow rate 310 ml/min and output from the measurement in 'mer02.mat'.

- Write an m-file and use equations (11) till (15).
- Optimize some parameters (mainly these which are hardly to measure) to minimize the deviation between model and real system steady states.

5.2.3 Task 3 – model from transfer function

- Create a Simulink model (mdl-file), which uses the process description in the form of transfer functions (equation (22)).
- The calculations of all coefficients for the transfer functions create in m-file.
- Verify this model with the measured data in file 'mer02.mat'.
- Optimize some parameters (mainly these which are hardly to measure) to minimize the deviation between model and real system dynamic behavior.
- Finally, do not forget to convert all signals to the physical units!

5.2.4 Task 4 – remote start of simulation

Expand the program from the task 3 so that the Simulink program (mdl-file) is started from the Matlab program (m-file).

5.2.5 Task 5 – data output

Expand the program from the task 4 so that the program (m-file) also includes graphical outputs from simulation.

5.2.6 Task 6 – state equation model

- Create a Simulink model (mdl-file) in the form of state equations (9), (10).
- Verify this model with the measured data in file 'mer02.mat'.
- Optimize some parameters (mainly these which are hardly to measure) to minimize the deviation between model and real system dynamic behavior.
- Compare advantages and disadvantages of both model types (transfer function v. state equations).

5.3 DISCUSSION OF THE RESULTS

The protocol must include mainly the following points.

1. Developed programs / models in the Matlab and Simulink.
2. Comparison of different forms of models.
3. Model verification and simulation results.
4. Time courses of the major inputs and outputs.
5. Discussion of the results.
6. Tables with parameters that result in a sufficient agreement of the real system with the model. Compare and discuss the changes of parameters values to obtain the same system behavior on the model as is measured in the file.

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