

Optimierung und Entscheidungsunterstützung (OuE)

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Topics for today:

- 1 CSP/CSOP
- 2 CSP/CSOP-Solver
- 3 Linear Programming

Repetition CSP

CSP

A Constraint Satisfaction Problem (V, D, C) consists of a set of variables V , their domains D and a set of constraints C on V .

CSP: Variable

- A Variable x is a placeholder for a value $\alpha(x)$.
- Each decision variable x has a domain D_x . It specifies the range of values the variable can take.

CSP: Constraint

- Constraints have a scope $S_c = \langle x_1, x_2, x_n \rangle$, where x_i is a variable.
- A constraint C is a relation defined on the domains of its scope $C \subseteq D_{x_1} \times D_{x_2} \times \dots \times D_{x_n}$

Constraints: Examples

Constraint

The start of meeting A must be later than the start of meeting B.

Constraints: Examples

The scope is:

Constraint

The **start of meeting A** must be later than the **start of meeting B**.

Constraints: Examples

The relation is:

Constraint

The start of meeting A must be **later than** the start of meeting B.

CSP: Solution

- An *instantiation* is a complete variable assignment (which does not necessarily satisfy all constraints).
- A *solution* is a variable assignment which satisfies all constraints.
- A *partial solution* is an assignment of a subset of variables which satisfies all constraints.

CSP: Solution

A **solution** is a variable assignment $\alpha : V \rightarrow D$, such that

- $\forall x \in V : \alpha(x) \in D_x$
- $\forall c \in C : S_c = \langle x_1, \dots, x_n \rangle \Rightarrow (\alpha(x_1), \dots, \alpha(x_n)) \in c$

Solving tasks for CSP

- Does a solution exist?
- Give me one solution or tell me that none exists!
- Give me all or several solutions!

CSOP

A (Constraint) Optimization Problem (CSOP/COP) is given by a CSP and an objective function $o : S \rightarrow M$, where

- S is the set of all solutions to the CSP
- M is a metric.

(typically M defined to be the set of real numbers)

CSOP

Solving tasks for CSOP/COP

- Find the best solution (if one exists)!
- Find the best solution you can get within a certain amount of time.

Basic solver for CSP

Generate-and-Test

```
assignment solve(csp){  
  
    while (not is_solution(csp, a)){  
        a = create_assignment(csp);  
    }  
    return a;  
}
```

Basic solver for CSP 2

Backtracking solver

```
back_solve(partial, variables, csp){  
    if empty(variables) return true  
  
    remove x from variables  
  
    for each value v in D_{x}{  
        extend partial to partial_neu with x <-- v  
  
        if valid(partial_neu, csp)  
            if back_solve(partial_neu, variables, csp)  
                return true  
    }  
    insert x in variables  
  
    return false  
}
```

Basic solver for CSOP

```
solution best;  
while( more_solutions_exist AND NOT interrupted){  
    s = createNewSolution()  
    if(o(s) < o(best)){  
        best = s;  
    }  
}
```

createNewSolution is a CSP-solver, that needs to deliver different (all?) solutions

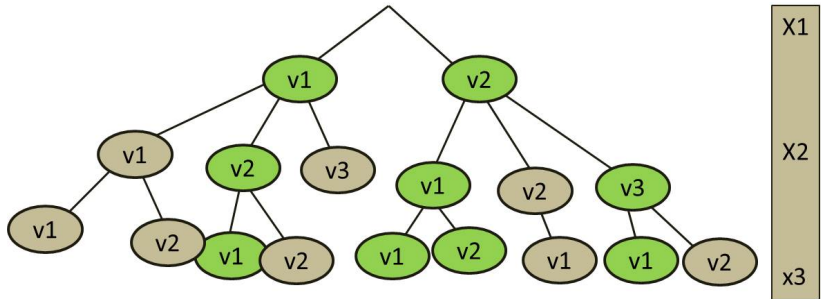
Basic solver for CSOP

Task

Modify backtracking-solver, such that it delivers *all* valid solutions.

Obtain all solutions

Backtracking solver



Obtain all solutions - BT solver

```
back_solve(partial, variables, csp){
```

```
  if empty(variables) return true
```

```
  remove x from variables
```

```
  for each value v in D_{x}{
```

```
    extend partial to partial_neu with x <-- v
```

```
    if valid(partial_neu, csp)
```

```
      if back_solve(partial_neu, variables, csp)
```

```
        return true
```

```
  }
```

```
  insert x in variables
```

```
  return false
```

```
}
```

At this point we have
a new solution

Do not cancel,
but proceed with
the remaining values in $D_{\{x\}}$!

Obtain all solutions - BT solver

```
back_solve(partial, variables, csp, all_solutions){  
  if empty(variables){  
    add partial to all_solutions  
  }else{  
    remove x from variables  
  
    for each value v in D_{x}{  
      extend partial to partial_neu with x <-- v  
  
      if valid(partial_neu, csp)  
        back_solve(partial_neu, variables, csp)  
    }  
  
    insert x in variables  
  }  
}
```

Complexity of CSP / CSOP

- The solution(s) of (V,D,C) are some of $|D|^{|V|}$ possible assignments.
- There is no polynomial algorithm known to solve the (general) CSP.
- The general CSP is NP-complete.

→ Naturally the CSOP is NP-complete because it includes a CSP

→ The problem is „harder” such that performance can be expected to be worse.

Excursion Complexity

Excursion Complexity

What we have learned:

- In general CSPs are NP-complete.
- For *general CSPs*, we have no algorithm for obtaining an (optimal) solution in reasonable time.
- However, for *practical CSPs*, our algorithms often are sufficient → not always need worst case time effort.
- Sometimes we can restrict the problem such that it belongs to P,
- or we use heuristics and approximate algorithms.

Modellierung - LP

Standardform:

minimiere/maximiere:

$$\sum_{j=1}^n c_j x_j \quad (1)$$

Bedingungen:

$$\sum_{j=1}^n a_{ij} x_j \approx b_i \quad i = 1, 2, \dots, m \quad (2)$$

$$x_j \geq 0 \quad j = 1, 2, \dots, n \quad (3)$$

$\approx$ means $=$, $\leq$ or $\geq$

Example 1

Fred's Coffee sells two blends of beans:Yusip Blend and Exotic Blend.

- Yusip Blend is **one-half Costa Rican beans** and **one-half Ethiopian beans**.
- Exotic Blend is **one-quarter Costa Rican beans** and **three-quarters Ethiopian beans**.
- Profit on the Yusip Blend is **\$3.50 per pound**, while profit on the Exotic Blend is **\$4.00 per pound**.
- Each day Fred receives a shipment of **200 pounds of Costa Rican beans** and **330 pounds of Ethiopian beans** to use for the two blends.

Question

How many pounds of each blend should be prepared each day to maximize profit? What is the maximum profit?

Example 1

Variables:

- x = amount of Yusip Blend
- y = amount of Exotic Blend

Maximize:

$$3.5 * x + 4 * y \quad (4)$$

Constraints:

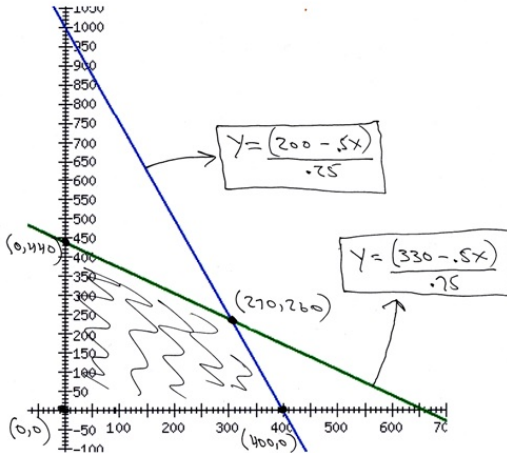
$$0.5 * x + 0.25 * y \leq 200$$

$$0.5 * x + 0.75 * y \leq 330$$

$$x \geq 0$$

$$y \geq 0$$

Example 1



Example 2

The Mapple store sells Mapple computers and printers.

- The computers are shipped in **12-cubic-foot boxes** and printers in **8-cubic-foot boxes**.
- The Mapple store estimates that at least 30 computers can be sold each month and that the number of computers sold will be at least 50% more than the number of printers.
- The computers cost the store \$1000 each and are sold for a profit of \$1000. The printers cost \$300 each and are sold for a profit of \$350.
- The store has a storeroom that can hold 1000 cubic feet and can spend \$70,000 each month on computers and printers.

Example 2

Question

How many computers and how many printers should be sold each month to maximize profit? What is the maximum profit?

Example 2

Variables:

- x = number of computers
- y = number of printers

Maximize:

$$1000 * x + 350 * y \quad (5)$$

Constraints:

$$x \geq 30$$

$$y \geq 1.5 * y$$

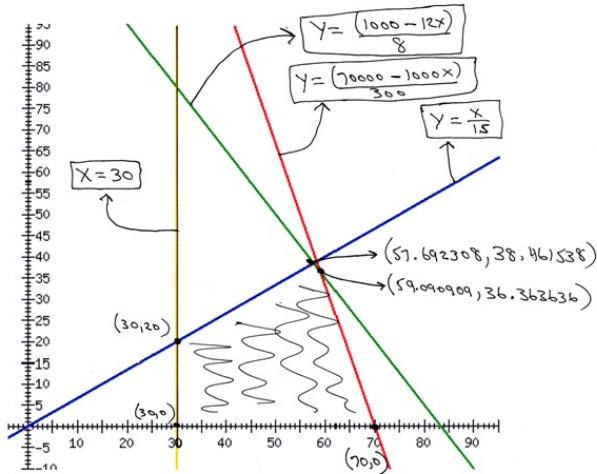
$$1000 * x + 300 * y \leq 70.000$$

$$12 * x + 8 * y \leq 1000$$

$$x \geq 0$$

$$y \geq 0$$

Example 2



Example 3

The Appliance Barn has 2400 cubic feet of storage space for refrigerators.

- Large refrigerators come in **60-cubic-foot packing crates** and small refrigerators come in **40-cubic-foot crates**.
- Large refrigerators can be sold for a **\$250 profit** and the smaller ones can be sold for **\$150 profit**.

Question

How many of each type of refrigerator should be sold to maximize profit and what is the maximum profit if:

a) At least 50 refrigerators must be sold each month.

Example 3

Variables:

- x = number of large refrigerators
- y = number of small refrigerators

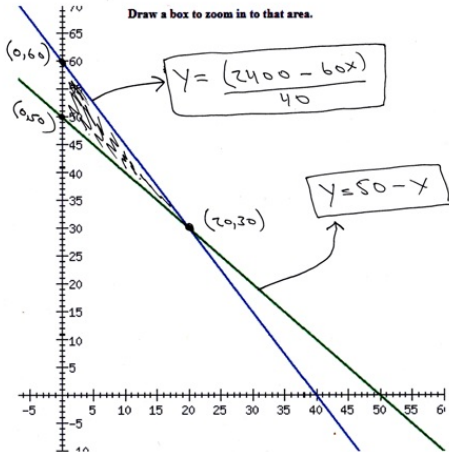
Maximize:

$$250 * x + 150 * y \quad (6)$$

Constraints:

$$\begin{aligned}x + y &\geq 50 \\60 * x + 40 * y &\leq 2400 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

Example 3



Example 4

Shannon's Chocolates produces **semisweet chocolate chips** and **milk chocolate chips** at its plants in Wichita, KS and Moore, OK.

- The Wichita plant produces 3000 pounds of semisweet chips and 2000 pounds of milk chocolate chips each day at a cost of \$1000,
- while the Moore plant produces 1000 pounds of semisweet chips and 6000 pounds of milk chocolate chips each day at a cost of \$1500
- Shannon has an order from Food Box Supermarkets for at least 30,000 pounds of semisweet chips and 60,000 pounds of milk chocolate chips.

Example 4

Question

How should Shannon schedule its production so that it can fill the order at minimum cost? What is the minimum cost?

Example 3

Variables:

- x = number of days of production at wichita plant
- y = number of days of production at moore plant.

Minimize:

$$1000 * x + 1500 * y \quad (7)$$

Constraints:

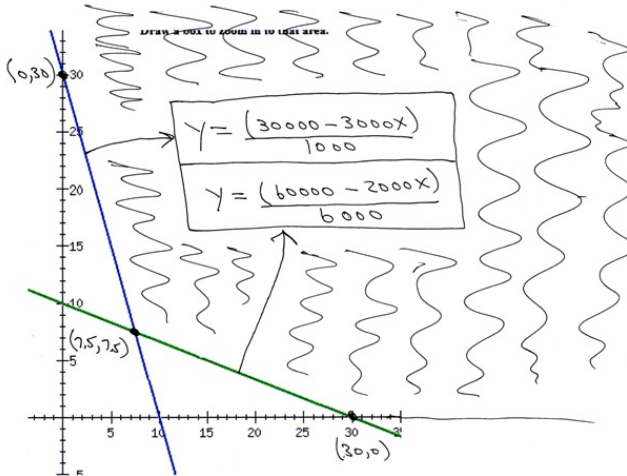
$$3000 * x + 1000 * y \geq 30000$$

$$2000 * x + 6000 * y \geq 60000$$

$$x \geq 0$$

$$y \geq 0$$

Example 4



Das war's für heute :-)