

Optimierung und Entscheidungsunterstützung (OuE)

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Topics for today:

- 1 LP Problems
- 2 IP and MIP Problems

Example 1

A cargo plane has three compartments for storing cargo: front, centre and rear. These compartments have the following limits on both weight and space:

Compartment	Weight capacity (tonnes)	Space capacity(cm)
Front	10	6800
Centre	16	8700
Rear	8	5300

Furthermore, the weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane.

Example 1

The following four cargoes are available for shipment on the next flight:

Cargo	Weight (tonnes)	Volume (cm/tonne)	Profit (£/tonne)
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

Any proportion of these cargoes can be accepted.

Example 1

Question

How much (if any) of each cargo C1, C2, C3 and C4 should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximised?

Example 1 - Solution

Variables:

- Let x_{ij} be the number of tonnes of cargo i ($i \in \{1, \dots, 4\}$ for C1, C2, C3 and C4 respectively) that is put into compartment $j \in \{1, \dots, 3\}$.
- $x_{ij} \geq 0$, $i \in \{1, \dots, 4\}$

Note here that we are explicitly told we can split the cargoes into any proportions (fractions) that we like.

Example 1 - Solution

Constraints:

cannot pack more of each of the four cargoes than we have available

- $x_{11} + x_{12} + x_{13} \leq 18$
- $x_{21} + x_{22} + x_{23} \leq 15$
- $x_{31} + x_{32} + x_{33} \leq 23$
- $x_{41} + x_{42} + x_{43} \leq 12$

Example 1 - Solution

Constraints:

the weight capacity of each compartment must be respected

- $x_{11} + x_{21} + x_{31} + x_{41} \leq 10$
- $x_{12} + x_{22} + x_{32} + x_{42} \leq 16$
- $x_{13} + x_{23} + x_{33} + x_{43} \leq 8$

Example 1 - Solution

Constraints:

the volume (space) capacity of each compartment must be respected

- $480x_{11} + 650x_{21} + 580x_{31} + 390x_{41} \leq 6800$
- $480x_{12} + 650x_{22} + 580x_{32} + 390x_{42} \leq 8700$
- $480x_{13} + 650x_{23} + 580x_{33} + 390x_{43} \leq 5300$

Example 1 - Solution

Constraints:

The weight of the cargo in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the plane.

$$[x_{11} + x_{21} + x_{31} + x_{41}]/10 = [x_{12} + x_{22} + x_{32} + x_{42}]/16 = [x_{13} + x_{23} + x_{33} + x_{43}]/8$$

Example 1 - Solution

Objective:

The objective is to maximise total profit, i.e.

$$\text{maximise } 310[x_{11} + x_{12} + x_{13}] + 380[x_{21} + x_{22} + x_{23}] + 350[x_{31} + x_{32} + x_{33}] + 285[x_{41} + x_{42} + x_{43}]$$

Example 1 - Solution

Constraints:

cannot pack more of each of the four cargoes than we have available for $i = 1$:

$$\sum_{j \in \{1, \dots, 3\}} x_{ij} \leq 18 \quad (1)$$

for $i = 2$:

$$\sum_{j \in \{1, \dots, 3\}} x_{ij} \leq 15 \quad (2)$$

etc.

Example 1 - MathProg

```
set Cargo;  
set Compartment;
```

```
param weight_cargo{i in Cargo};  
param volume_cargo{i in Cargo};  
param profit_cargo{i in Cargo};
```

```
param w_capacity_compartment{j in Compartment};  
param v_capacity_compartment{j in Compartment};
```

```
var x{i in Cargo, j in Compartment}  $\geq$  0;
```

Example 1 - MathProg

s.t. amount_cargo{i in Cargo}:

$$\text{sum}\{j \text{ in Compartment}\} x[i,j] \leq \text{weight_cargo}[i];$$

s.t. weight_capacity{j in Compartment}:

$$\text{sum}\{i \text{ in Cargo}\} x[i,j] \leq \text{w_capacity_compartment}[j];$$

s.t. volume_capacity{j in Compartment}:

$$\begin{aligned} &\text{sum}\{i \text{ in Cargo}\} \text{volume_cargo}[i] * x[i,j] \\ &\leq \text{v_capacity_compartment}[j]; \end{aligned}$$

Example 1 - MathProg

s.t. proportion{j in **Compartment**: j $\geq$ 2}:

$$\text{sum}\{i \text{ in Cargo}\} (1/w_capacity_compartment[j])*x[i,j] = \\ \text{sum}\{i \text{ in Cargo}\} (1/w_capacity_compartment[j-1])*x[i,j-1];$$

maximize profit: sum{i in Cargo, j in Compartment}
profit_cargo[i]*x[i,j];

Das war's für heute :-)