

Optimierung und Entscheidungsunterstützung (OuE)

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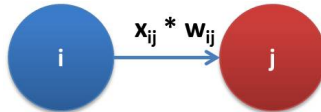
Topics for today:

1 Network Flow Programming

2 Constraint Programming

Network Flow Programming

- Network modeling can be used if there are (binary, integer) variables x_{ij} that express a relationship between two entities i and j .
- Nodes of the network denote entities.
- Arcs of the network denote relationships $\rightarrow$ often weights w_{ij} are assigned to the arcs.



Network Flow Programming

Typical network flow constraints:

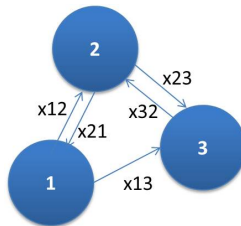
- **outflow constraints:** $\sum_{j \in \{1, \dots, N\}} x_{ij} = a_i$
- **inflow constraints:** $\sum_{i \in \{1, \dots, N\}} x_{ij} = b_j$
- **flow conservation constraints:** e.g.

$$\sum_{i \in \{1, \dots, N\}} x_{ij} = \sum_{k \in \{1, \dots, N\}} x_{jk}$$
- **capacity constraints:** e.g. $x_{ij} \leq c_{ij}$, if x_{ij} is integer

Typical optimization goal:

- $\min \sum_{i,j \in \{1, \dots, N\}} x_{ij} * w_{ij}$

Networks with binary variables x_{ij}



Networks with binary variables x_{ij}

- binary variables turn arcs on (1) or off (0)
- **Examples:** Single Source Shortest Path Problem (SSSP), Job Shop Scheduling Problem (JSSP), Traveling Salesman Problem (TSP)

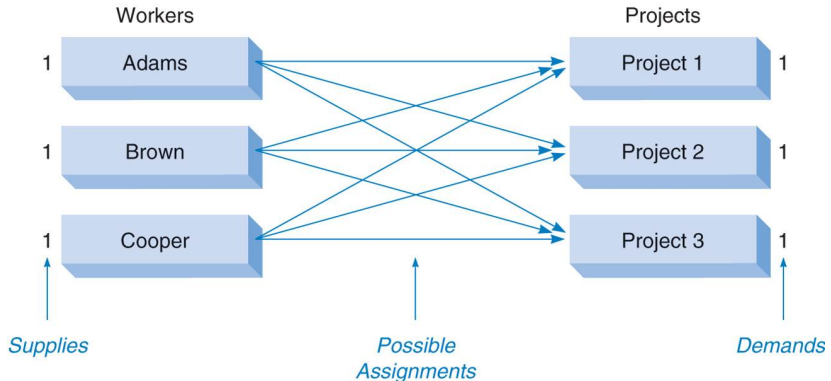
Assuming, there are N entities:

- *It depends on the problem*, how many variables $x_{ij}, i, j \in \{1, \dots, N\}$ are set up.
- *It depends on the problem*, how the solution graph consisting of *some* arcs of the network, should look like
- solution graph is described with constraints on the binary variables

Assignment problem

Assignment problem

Assign workers $i \in \{1, \dots, W\}$ to projects $j \in \{1, \dots, P\}$, such that the costs are minimal. Each worker needs exactly one project and each project needs exactly one worker.



Assignment problem

Estimated Wages Cost of Possible Assignments

TABLE 5.3

Estimated Project Repair
Costs for Fix-It Shop

PERSON	PROJECT		
	1	2	3
Adams	\$11	\$14	\$ 6
Brown	\$ 8	\$10	\$11
Cooper	\$ 9	\$12	\$ 7

Networks with integer variables x_{ij}

- Integer variables x_{ij} typically represent goods or persons that flow through a network.

Maximum Flow Problem

Find a feasible flow through a single-source, single-sink flow network that is maximum.

Examples:

- How much water can flow through a network of pipes?
- How many cars can travel through a network of streets?

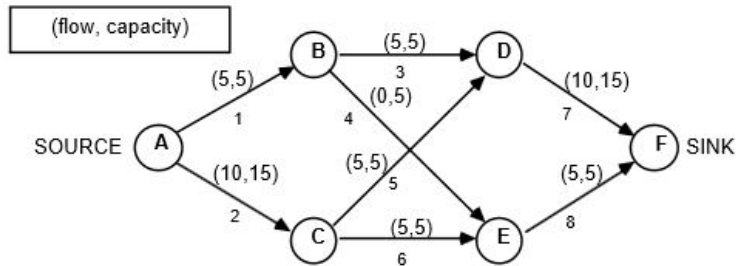


Figure 15. Example maximum flow problem with solution

Maximum Flow Problem

Let $N = (V, E)$ be a network with $s, t \in V$ being the source and the sink of N respectively.

- The **capacity** of an edge is a mapping $c : E \rightarrow \mathbb{R}^+$, denoted by c_{uv} . It represents the maximum amount of flow that can pass through an edge.
- A **flow** is a mapping $x : E \rightarrow \mathbb{R}^+$, denoted by x_{uv} , subject to the following constraints:

Capacity constraints: $x_{uv} \leq c_{uv}$, for each $(u, v) \in E$

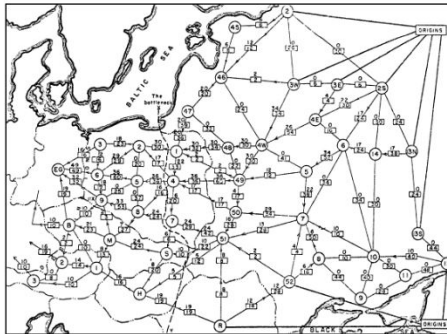
Flow conservation: $\sum_{u:(u,v) \in E} x_{uv} = \sum_{u:(v,u) \in E} x_{vu}$, for each $v \in V \setminus \{s, t\}$

- The **value of flow** is defined by $|f| = \sum_{v:(s,v) \in E} x_{sv}$, where s is the source of N . It represents the amount of flow passing from the source to the sink.

Maximum Flow Problem

Find the Maximum Flow

The maximum flow problem is to maximize $|f|$, that is, to route as much flow as possible from s to t .



Harris and Ross's map of the Warsaw Pact rail network

Maximum Flow Problem

One of the first recorded applications

In the mid-1950s, Air Force researchers T. E. Harris and F. S. Ross published a classified report studying the rail network that linked the Soviet Union to its satellite countries in Eastern Europe. The network was modeled as a graph with 44 vertices, representing geographic regions, and 105 edges, representing links between those regions in the rail network. Each edge was given a weight, representing the rate at which material could be shipped from one region to the next. Essentially by trial and error, they determined both the maximum amount of stuff that could be moved from Russia into Europe, as well as the cheapest way to disrupt the network by removing links (or in less abstract terms, blowing up train tracks), which they called 'the bottleneck'. Their results (including the figure at the top of the page) were only declassified in 1999.

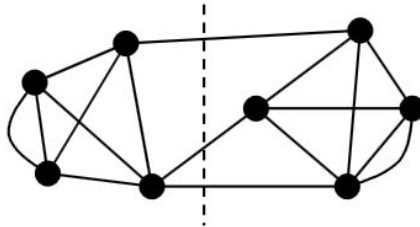
Maximum Flow Problem

Solution Strategies:

http://en.wikipedia.org/wiki/Maximum_flow_problem

Minimum Cut Problem

Given a graph $G = (V, E)$, V the vertices, E the set of edges of G , the minimum cut is the smallest set of edges whose removal breaks G into two pieces.



Minimum Cut Problem - Application

- Finding the minimum cut in a graph plays an important role in the design of communication networks,
- If enough links fail, there will be at least one pair of nodes that cannot communicate with each other.
- Thus an important measure of the reliability of a network is the minimum number of links that must fail in order for this to happen.

Das war's für heute :-)