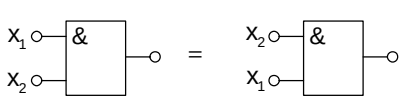
Die Boolsche Schaltalgebra

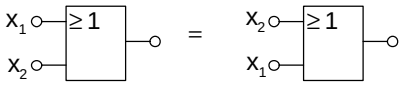
· Rechengesetze, die zur Schaltungsvereinfachung benutzt werden:

(0) · bindet stärker als $\vee$: $x_1 \vee x_2 \cdot x_3 = x_1 \vee (x_2 \cdot x_3)$

(1) $x_1 \cdot x_2 = x_2 \cdot x_1$

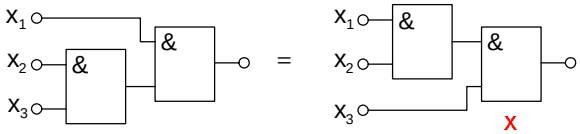


(2) $x_1 \vee x_2 = x_2 \vee x_1$

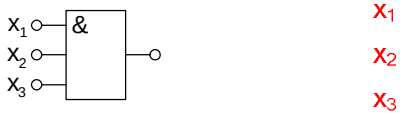


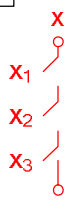
Kommutativ-gesetze

(3) $x_1 \cdot (x_2 \cdot x_3) = (x_1 \cdot x_2) \cdot x_3$



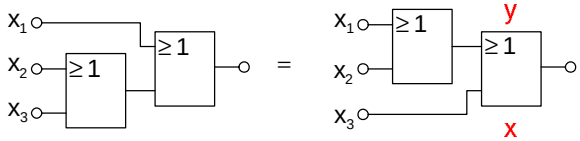
$= x_1 \cdot x_2 \cdot x_3$



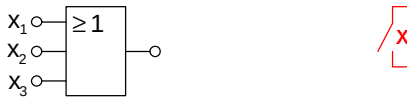


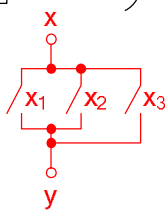
Assoziativ-gesetze

(4) $x_1 \vee (x_2 \vee x_3) = (x_1 \vee x_2) \vee x_3$



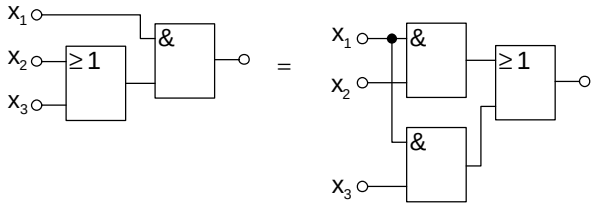
$= x_1 \vee x_2 \vee x_3$



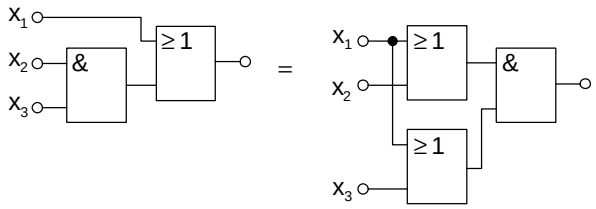


$\Rightarrow$ UND-, ODER, NAND- und NOR - Gatter sind mit mehr als 2 Eingängen möglich

(5) $x_1 \cdot (x_2 \vee x_3) = (x_1 \cdot x_2) \vee (x_1 \cdot x_3)$

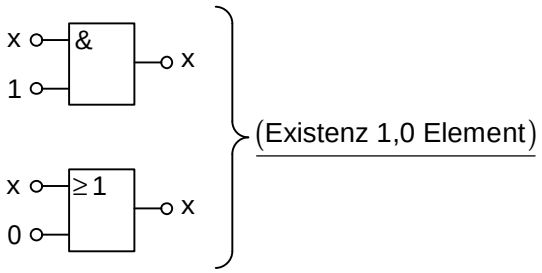


(6) $x_1 \vee (x_2 \cdot x_3) = (x_1 \vee x_2) \cdot (x_1 \vee x_3)$



Distributiv-gesetze

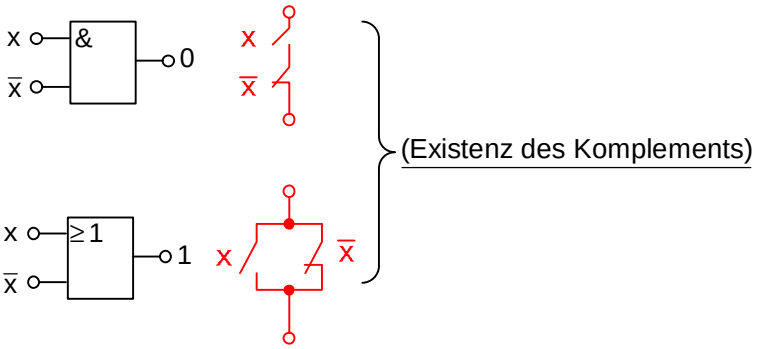
(7) $x \cdot 1 = x$



(Existenz 1,0 Element)

(8) $x \vee 0 = x$

(9) $x \cdot \overline{x} = 0$



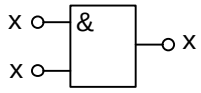
(Existenz des Komplements)

(10) $x \vee \overline{x} = 1$

- Alle anderen Regeln lassen sich aus den Regeln (0) .. (10) ableiten (beweisen)
- Regel (0) .. (10) werden deshalb als Axiome bezeichnet.

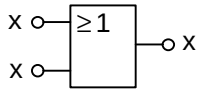
(11) $x \cdot x = x$

<u>Beweis:</u> $x \cdot x \overset{(8)}{=} (x \cdot x) \vee 0 \overset{(9)}{=} (x \cdot x) \vee (x \cdot \overline{x}) \overset{(5)}{=} x \cdot (x \vee \overline{x}) \overset{(10)}{=} x \cdot 1 \overset{(7)}{=} x$



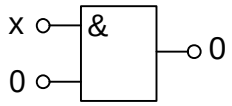
(12) $x \vee x = x$

<u>Beweis:</u> $x \vee x \overset{(7)}{=} (x \vee x) \cdot 1 \overset{(10)}{=} (x \vee x) \cdot (x \vee \overline{x}) \overset{(6)}{=} x \vee (x \cdot \overline{x}) \overset{(9)}{=} x \vee 0 \overset{(8)}{=} x$

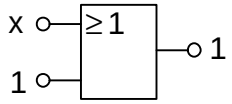


(Idempotenzgesetze)

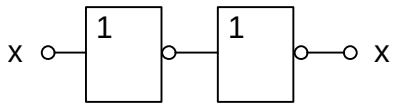
(13) $x \cdot 0 \overset{(2)}{=} 0 \cdot x = 0$



(14) $x \vee 1 \overset{(2)}{=} 1 \vee x = 1$



(15) $\overline{\overline{x}} = x$



(16) $\overline{x_1 \cdot \ldots \cdot x_n} = \overline{x}_1 \vee \ldots \vee \overline{x}_n$

z.B. $\overline{x_1 \cdot x_2} = \overline{x}_1 \vee \overline{x}_2$

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

(De-Morgansche Regeln)

(17) $\overline{x_1 \vee \ldots \vee x_n} = \overline{x}_1 \cdot \ldots \cdot \overline{x}_n$

z.B. $\overline{x_1 \vee x_2} = \overline{x}_1 \cdot \overline{x}_2$

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

allgemein:

(18) $\overline{f(x_1,\dots,x_n;0,1;\cdot,\vee)} = f(\overline{x}_1,\dots,\overline{x}_n;1,0;\vee\cdot)$

(Shannonsches Gesetz)

z.B. De-Morgansche Regeln (16) und (17)

$f(x_1,x_2;\cdot,\vee) = x_1 \cdot x_2$ $\qquad$ $\overline{f(x_1,x_2;\cdot,\vee)} = \overline{x_1 \cdot x_2} = f(\overline{x}_1,\overline{x}_2;\vee,\cdot) = \overline{x}_1 \vee \overline{x}_2$

(19) $f(x_1,\dots,x_i,\dots,x_n) = x_i \cdot f(x_1,\dots,1,\dots,x_n) \vee \overline{x}_i \cdot f(x_1,\dots,0,\dots,x_n)$

(Entwicklungssatz)

(20) Die duale Schaltfunktion f^D einer Schaltfunktion f erhält man, in dem man UND- und ODER-Operationen mit einander vertauscht:

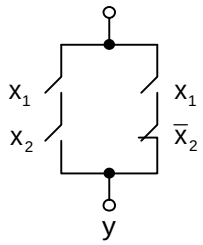
$f^D(x_1,\dots,x_n;0,1;\cdot,\vee) = f(x_1,\dots,x_n;1,0;\vee,\cdot)$

(Dualitätsgesetz)

Kürzungsregeln

- Anwendung der Rechengesetze der Boolschen Schaltalgebra zur Vereinfachung von Schaltfunktionen:

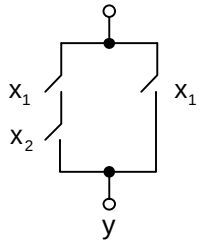
(K1) $\underline{\underline{x_1 \cdot x_2 \vee x_1 \overline{x}_2}} \overset{(5)}{=} x_1 \cdot (x_2 \vee \overline{x}_2) \overset{(10)}{=} x_1 \cdot 1 \overset{(7)}{=} \underline{\underline{x_1}}$



2 ODER-verknüpfte Konjunktionen, die in allen Variablen übereinstimmen bis auf genau eine, die einmal einfach und das andere mal negiert vorkommt.

(K2) $\underline{\underline{x_1 \vee x_1 \cdot x_2}} \overset{(5)}{=} x_1 \cdot (\underbrace{1 \vee x_2}_{=1})$

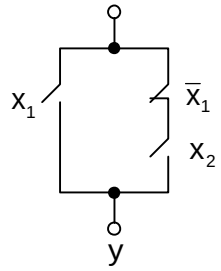
$= x_1 \cdot 1 \overset{(7)}{=} \underline{\underline{x_1}}$



(K3) $\underline{\underline{x_1 \vee \overline{x}_1 \cdot x_2}} \overset{(0)}{=} x_1 \vee (\overline{x}_1 \cdot x_2) \overset{(6)}{=} (x_1 \vee \overline{x}_1) \cdot (x_1 \vee x_2) \overset{(10)}{=} 1 \cdot (x_1 \vee x_2) \overset{(7)}{=} \underline{\underline{x_1 \vee x_2}}$

$x_1 \vee \overline{x}_1 \cdot x_2 \cdot x_3 = x_1 \vee x_2 \cdot x_3$

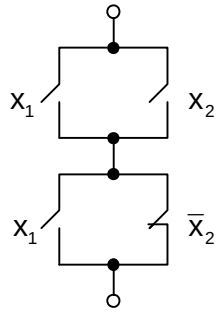
kommt als Faktor in anderem Term negiert vor



(K4) duale Form von (K1)

$((x_1 \cdot x_2) \vee (x_1 \cdot \overline{x}_2))^D = (x_1)^D$

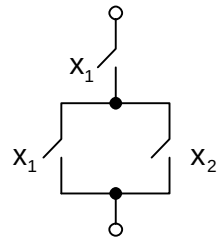
$\underline{\underline{(x_1 \vee x_2) \cdot (x_1 \vee \overline{x}_2)}} = \underline{\underline{x_1}}$



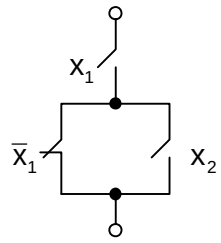
(K5) duale Form von (K2)

$(x_1 \vee (x_1 \cdot x_2))^D = (x_1)^D$

$\underline{\underline{x_1 \cdot (x_1 \vee x_2)}} = \underline{\underline{x_1}}$



(K6) duale Form von (K3)

$(x_1 \vee (\overline{x}_1 \cdot x_2))^D = (x_1 \vee x_2)^D$

$\underline{\underline{x_1 \cdot (\overline{x}_1 \vee x_2)}} = \underline{\underline{x_1 \cdot x_2}}$